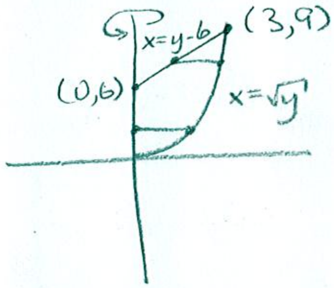
Math 1B
Quiz 4 Version A
Fri Oct 26, 2012

NAME YOU ASKED TO BE CALLED IN CLASS: ______________________

SCORE: _____ / 30 POINTS

NO CALCULATORS ALLOWED

The region bounded in the first quadrant by $y = x^2$, $y = x + 6$ and $x = 0$ is revolved around the $y-$axis. **SCORE: _____ / 6 PTS**
Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid using the disk or washer method.



$$\pi \int_0^6 (\sqrt{y})^2\, dy + \pi \int_6^9 ((\sqrt{y})^2 - (y-6)^2)\, dy$$

$$= \pi \int_0^6 y\, dy + \pi \int_6^9 (y - (y-6)^2)\, dy$$

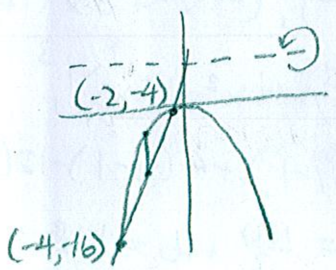
$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1 1 $\frac{1}{2}$

$x^2 = x + 6$
$x^2 - x - 6 = 0$
$(x-3)(x+2) = 0$
$x = 3, \cancel{-2}$

MINUS $\frac{1}{2}$ POINT IF YOU FORGOT THIS dy
(BUT NOT IF YOU FORGOT THIS ENTIRE INTEGRAL FROM $y=0$ TO $y=6$)

The region bounded by $y = -x^2$ and $y = 6x + 8$ is revolved around the line $y = 2$. **SCORE: _____ / 9 PTS**

[a] Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid using the disk or washer method.



$$\pi \int_{-4}^{-2} ((2-(6x+8))^2 - (2 - -x^2)^2)\, dx$$

$$= \pi \int_{-4}^{-2} ((-6x-6)^2 - (2+x^2)^2)\, dx$$

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1 1 $\frac{1}{2}$

$-x^2 = 6x + 8$
$0 = x^2 + 6x + 8$
$0 = (x+2)(x+4)$
$x = -2, -4$

[b] Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid using the shell method.

$y = -x^2 \rightarrow x = \pm\sqrt{-y}$
$x = -\sqrt{-y}$

$y = 6x + 8 \rightarrow x = \frac{1}{6}(y-8)$

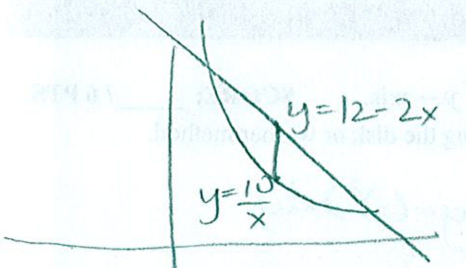
$$2\pi \int_{-16}^{-4} (2-y)\left(\tfrac{1}{6}(y-8) - -\sqrt{-y}\right) dy$$

$$= 2\pi \int_{-16}^{-4} (2-y)\left(\tfrac{1}{6}(y-8) + \sqrt{-y}\right) dy$$

$\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ 1 1 1 $\frac{1}{2}$

The base of a solid is the region bounded by $y = \frac{10}{x}$ and $y = 12 - 2x$. Cross sections perpendicular to the $x-$axis are equilateral triangles. Write, **BUT DO NOT EVALUATE**, an integral (or sum of integrals) for the volume of the solid.

SCORE: _____ / 5 PTS



$$\frac{\sqrt{3}}{4}\int_1^5 \left(12-2x-\frac{10}{x}\right)^2 dx$$

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$\frac{10}{x} = 12-2x$

$10 = 12x - 2x^2$

$2x^2 - 12x + 10 = 0$

$2(x-1)(x-5) = 0 \rightarrow x = 1, 5$

Find the area between the curves $y = -3x^2 - 9x$ and $y = x^2 - x - 12$ over the interval $0 \le x \le 3$.

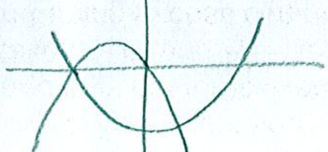
SCORE: _____ / 6 PTS

$-3x^2 - 9x = x^2 - x - 12$

$0 = 4x^2 + 8x - 12$

$0 = 4(x+3)(x-1)$

$x = -3, 1$



$$\int_0^1 (-3x^2-9x-(x^2-x-12))\,dx + \int_1^3 (x^2-x-12-(-3x^2-9x))\,dx$$

$$= \int_0^1 (-4x^2-8x+12)\,dx + \int_1^3 (4x^2+8x-12)\,dx$$

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$$= \left(-\frac{4}{3}x^3 - 4x^2 + 12x\right)\Big|_0^1 + \left(\frac{4}{3}x^3 + 4x^2 - 12x\right)\Big|_1^3$$

(1) (1)

$$= -\frac{4}{3}(1) - 4(1) + 12(1) + \frac{4}{3}(27-1) + 4(9-1) - 12(3-1)$$

$$= -\frac{4}{3} - 4 + 12 + \frac{104}{3} + 32 - 24 = \frac{100}{3} + 16 = \frac{148}{3}$$

(1)

A solid is created by revolving a region around an axis of revolution. Sketch the region and find the equation of the axis of revolution if the volume of the solid is $\pi \int_0^1 ((4 - e^y)^2 - (4 + y^2)^2)\,dy$.

SCORE: _____ / 4 PTS

WASHER METHOD

HORIZONTAL CUT ⊥ VERTICAL AXIS $x = 4$

$x = e^y \rightarrow y = \ln x$

$x = -y^2$